NATURE OF THE STATE OF THE MEDIUM IN THE NEIGHBORHOOD OF A CAVITY EXPANDING INTO A DILATING MEDIUM

S. Z. Dunin, V. K. Sirotkin, and E. V. Sumin

UDC 539.374

Substantial changes in the state of a solid medium can occur under the expansion of a gas cavity therein. In particular, rupture of the brittle rock occurs. The nature of the motion of the ruptured rock differs substantially from the nature of the motion of the unruptured medium. Thus, a change in the density of the ruptured rock occurs under shear strains. This phenomenon is usually called dilatancy [1]. In addition, the strength characteristics also change under rupture of the rock. The stress state of a medium in the neighborhood of an expanding cavity at the time of cessation is analyzed in this paper. The influence of the ruptured rock characteristics on the magnitude of the residual stress is investigated. The radius of the rupture zone is determined and its dependence on the characteristics of the medium is investigated. The volume of the threshold space in the neighborhood of the cavity being formed because of dilatancy is calculated. The nature of the stress state in elastic–plastic media which do not dilate under plastic flow is also investigated.

1. Let us examine adiabatic expansion of a spherical gas cavity in an elastic–plastic medium. A shock wave in which the stress exceeds the strength of the medium under crushing, starts to be propagated at the initial instant in an elastic–plastic medium; i.e., the shock front agrees with the rupture front. It is assumed that compression of the medium by a certain constant magnitude ε independent of the wave intensity [2]

$$\varepsilon = (\rho - \rho_0)/\rho = \text{const}$$

occurs on the shock front.

Plastic flow of the ruptured medium loosened (compressed) because of dilatancy occurs between the expanding cavity and the shock front. We assume that the rate of dilatancy Λ is constant in the whole plastic flow domain. The plastic flow is described by the Coulomb–Mohr fluidity condition [3]

$$|\sigma_r - \sigma_\varphi| = -\sin\varphi(\sigma_r + \sigma_\varphi) + 2c\cos\varphi,$$

which under expansion of the cavity acquires the form

$$(1 + \alpha)\sigma_\varphi - \sigma_r - Y = 0, \tag{1.1}$$

where

$$\alpha = 2\sin\varphi/(1 - \sin\varphi);\ Y = 2c\cos\varphi/(1 - \sin\varphi).$$

The flow of a granulated medium behind a shock front will be described by the equation of motion

$$\rho(r, t)\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) = \frac{\partial\sigma_r}{\partial r} + 2\frac{\sigma_r - \sigma_\varphi}{r}, \tag{1.2}$$

the fluidity condition (1.1), and the dilatancy condition [1]

$$I_1 = 2\Lambda\sqrt{I_2}. \tag{1.3}$$

For spherically symmetric motion and Λ=const, Eq. (1.3) becomes

$$\frac{\partial v}{\partial r} + 2\frac{v}{r} = \Lambda\left|\frac{\partial v}{\partial r} - \frac{v}{r}\right|. \tag{1.4}$$

Integration of condition (1.4) determines the velocity field in the plastic zone under expansion of the cavity:

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 143-152, May-June, 1979. Original article submitted January 5, 1978.

$$v(r,t)=\frac{\lambda_p(t)}{r^n},\quad n=\frac{2-\Lambda}{1+\Lambda}. \tag{1.5}$$

Within the framework of this model, an expression has been obtained in [4] for the soil density while taking account of the compression on the shock front and the loosening because of dilatancy:

$$\rho(r,t)=\rho_0\left[\frac{1-\left(\frac{a}{r}\right)^{n+1}}{1-\varepsilon}+\left(\frac{a_0}{r}\right)^{n+1}\right]^{\Lambda}(1+\varepsilon), \tag{1.6}$$

where a_0, a are the initial and running radii of the cavity, ρ_0 is the initial density, and n is determined from (1.5).

Substituting (1.1), (1.5), (1.6) into (1.2), we obtain

$$\rho(r,t)\left(\frac{\dot{\lambda}_p}{r^n}-n\frac{\lambda_p^2}{r^{2n+1}}\right)=\frac{\partial\sigma_r}{\partial r}+\frac{2\alpha}{\alpha+1}\frac{\sigma_r}{r}-\frac{2Y}{\alpha+1}\frac{1}{r}, \tag{1.7}$$

$$\sigma_\varphi=(\sigma_r+Y)/(1-\alpha).$$

Integrating (1.7) between a and r, we obtain

$$\sigma_r(r)=C(t)r^{-\omega}+r^{-\omega}\dot{\lambda}_p a^{\omega-n+1}F_1(r)-r^{-\omega}n\lambda_p^2 a^{\omega-2n}F_2(r)+\frac{2Y}{\omega(1+\alpha)}\left\{1-\left(\frac{a}{r}\right)^{\omega}\right\},\quad \omega=\frac{2\alpha}{\alpha+1}; \tag{1.8}$$

$$F_1(r)=\rho_0(1+\varepsilon)\int\limits_{a/r}^{1}\left[\frac{1-\xi^{n+1}}{1-\varepsilon}+\left(\frac{a_0}{a}\right)^{n+1}\xi^{n+1}\right]^{\Lambda}\xi^{n-\omega-2}d\xi, \tag{1.9}$$

$$F_2(r)=\rho_0(1+\varepsilon)\int\limits_{a/r}^{1}\left[\frac{1-\xi^{n+1}}{1-\varepsilon}+\left(\frac{a_0}{a}\right)^{n+1}\xi^{n+1}\right]^{\Lambda}\xi^{2n-\omega-1}d\xi,$$

where $\xi=a/r$.

The quantities C (t), $\dot{\lambda}_p(t)$, $\lambda_p(t)$ are found according to the boundary condition on the cavity wall

$$\sigma_r|_{r=a}=-p(a)=-p_0\left(\frac{a_0}{a}\right)^{3\gamma}, \tag{1.10}$$

where γ is the adiabatic index, and the boundary condition on the rupture front. Using (1.10), we find

$$C(t)=-p(a)a^{\omega}.$$

At the end of cavity expansion the shock damps because of dissipative processes and degenerates into an elastic predecessor; i.e., a dynamic waveless expansion of the cavity sets in [5]. Even at this stage, the rupture front radiates elastic waves. The propagation equation for small perturbations in an elastic medium has the form

$$\operatorname{grad}\operatorname{div}\mathbf{v}=\frac{1}{c_0^2}\frac{\partial^2\mathbf{v}}{\partial t^2}.$$

Let us consider that $c_0 \gg \dot{a}$ at the stage of waveless expansion of the cavity. Then the elastic domain can be considered an incompressible medium. The velocity field in the elastic domain has the form

$$v(r,t)=\lambda_e(t)/r^2,\ \lambda_e=a^2\dot{a}. \tag{1.11}$$

The elastic strains are described by the equation of motion

$$\rho_0(\partial v/\partial t+v\partial v/\partial r)=\partial\sigma_r/\partial r+2(\sigma_r-\sigma_\varphi)/r \tag{1.12}$$

and the convective Hooke relationships [6]

$$E\partial v/\partial r=\dot{\sigma}_r-\dot{\sigma}_\varphi,\quad Ev/r=(1/2)(\dot{\sigma}_\varphi-\dot{\sigma}_r), \tag{1.13}$$

where E is Young's modulus and the point denotes the total derivative with respect to time. Using (1.11) and (1.13), we obtain

$$\dot{\sigma}_\varphi-\dot{\sigma}_r=2E\lambda_e/r^3,\ \sigma_r-\sigma_\varphi=2E\ln(1-u/r), \tag{1.14}$$

where u is the displacement of a material point.

In order to determine the magnitude of the shift into the elastic domain, let us use the equation of mass balance

$$\int_{a}^{r} \rho(r, t) r^2 dr = \int_{a_0}^{r-u} \rho(r, t) r^2 dr, \quad r - u > R, \tag{1.15}$$

where R is the radius of the rupture front. We finally obtain from (1.6), (1.14) and (1.15)

$$\sigma_r - \sigma_\varphi = \frac{2}{3} \mathrm{E} \ln\left\{1 - \frac{1}{r^3}\left[R^3 - a_0^3 - 3a^3 F_3(R)\right]\right\} \equiv \frac{2}{3} \mathrm{E} \ln\left[1 - \frac{g(R)}{r^3}\right], \tag{1.16}$$

where

$$F_3(R) = (1 + \varepsilon) \int_{a/R}^{1} \left[\frac{1 - \xi^{n+1}}{1 - \varepsilon} + \left(\frac{a_0}{a}\right)^{n+1} \xi^{n+1}\right]^{\Lambda} \frac{d\xi}{\xi^4}. \tag{1.17}$$

Then the solution of (1.12), taking account of the boundary condition for the stress at infinity

$$\sigma_r(r) \to -p_h, \tag{1.18}$$

where p_h is the lithostatic pressure, has the form

$$\sigma_r(r) = -p_h - \frac{4\mathrm{E}}{9} \mathrm{Li}_2\left[\frac{g(R)}{r^3}\right] - \rho_0\left(\frac{\dot{\lambda}_e}{r} - \frac{\lambda_e^2}{2r^4}\right),$$

where

$$\mathrm{Li}_2(x) = -\int_0^x \ln(1 - \eta) \frac{d\eta}{\eta}.$$

The solutions in the elastic and plastic domains should be "joined" at the rupture front. The conditions on the rupture front have the form

$$\sigma_r(R - 0) = \sigma_r(R + 0), \; \lambda_e/r^2 = \lambda_p/r^n \tag{1.19}$$

to the end of cavity expansion.

2. Let us examine the behavior of σ_r in the elastic and plastic domains up to the end of cavity expansion; in this case $\lambda_e = \lambda_p = 0$, $(a_0/a_m)^{n+1} \ll 1$, $\varepsilon \ll 1$, where a_m is the radius of the cavity at the time of cancellation. Since the integral in (1.17) is determined by small ξ, then by neglecting the small domain $1 - \xi^{n+1} \lessgtr (a_0/a_m)^{n+1}$ for the integrand, the approximate expression can be used:

$$\varphi_3(\xi) = \left[1 - \xi^{n+1}\right]^{\Lambda} \frac{1}{\xi^4} \approx \left(1 - \Lambda\xi^{n+1}\right) \frac{1}{\xi^4}, \quad \text{if} \quad \frac{1 - \Lambda}{2} \xi^{n+1} \ll 1.$$

This latter equality is valid down to $\xi = 0.9$; i.e., $r = 1.1a_m$. A graph of the function $\varphi_3(\xi) = (\Lambda = 0.1; n = 1.7)$ from which it is seen that the integrand $\varphi_3(\xi)$ yields a very small contribution in the interval [0.8, 1], is given in Fig. 1. Hence, the expression

$$F_3(R) = \int_{a/R}^{1} \left(1 - \Lambda\xi^{n+1}\right) \frac{d\xi}{\xi^4} = \frac{1}{3}\left[\left(\frac{R}{a}\right)^3 - \left(\frac{R}{a}\right)^{3\Lambda}\right]$$

is valid for the integral (1.17).

The volume replaced by the spherical cavity during the expansion of an incompressible plastic medium equals $a_m^3 - a_0^3$. In our case, loosening (compression) occurs in the plastic zone because of dilatancy. Hence the volume displaced by the cavity equals

$$g(R_m) = a_m^3 \left(\frac{R_m}{a_m}\right)^{3\Lambda} - a_0^3. \tag{2.1}$$

The volume converted into pore formation during loosening ($\Lambda > 0$) equals

$$V_p = a_m^3 \left[\left(\frac{R_m}{a_m}\right)^{3\Lambda} - 1\right].$$

Substituting (2.1) into (1.16), we obtain the solution in the elastic domain

$$-\sigma_r(r) = p_h + \frac{4\mathrm{E}}{9r^3} g(R_m) + \rho_0 \frac{\dot{\lambda}_e}{r}. \tag{2.2}$$

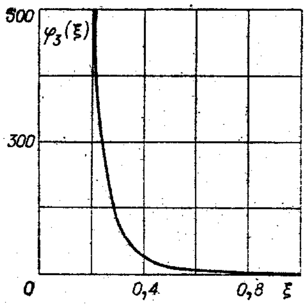

Fig. 1

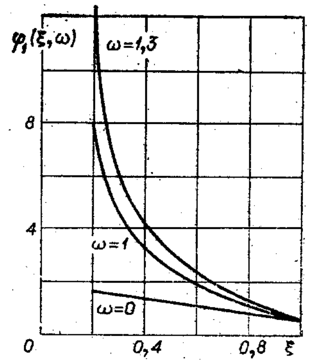

Fig. 2

Here the expansion $\ln(1-\eta)\approx-\eta$ at $\eta\ll1$ was used. As is seen from (2.2), the radial stresses can reach a minimum at the point r_{**}:

$$r_{**}=\left[\frac{4E}{3\rho_0|\dot\lambda_e|}g(R_m)\right]^{\frac{1}{2}}.$$

If loosening occurs in the plastic domain, then r_{**} is shifted from the cavity.

Let us turn to an analysis of the plastic domain. To obtain the solution in the plastic domain, the integrand $\varphi_1(\xi,\omega)$ of the integral (1.9), whose graph is shown in Fig. 2, must be investigated. Let us expand $\varphi_1(\xi,\omega)$ in powers of Λ and let us limit ourselves to a linear term. Expanding $\ln(1-\xi^{n+1})$ in series, we obtain the following expression

$$F_1(r)=\rho_0\int_{a_m/r}^{1}\xi^{n-\omega-2}\,d\xi-\Lambda\rho_0\sum_{k=1}^{\infty}\int_{a_m/r}^{1}\xi^{k(n+1)+n-\omega-2}\,d\xi=$$

$$=\frac{\rho_0}{\omega-n+1}\left[\left(\frac{a_m}{r}\right)^{\omega-n+1}-1\right]-\Lambda\rho_0\left[\sum_{k=1}^{\infty}\frac{1}{k(n+1)+n-\omega-1}-\sum_{k=1}^{\infty}\frac{\left(\frac{a_m}{r}\right)^{k(n+1)+n-\omega-1}}{k(n+1)+n-\omega-1}\right].$$

Retaining just the main terms in these series, we obtain for σ_r at the time of stopping during expansion of the cavity (for $r>1.1a_m$)

$$\sigma_r(r)=\frac{Y}{\alpha}-\left[p(a_m)+\frac{Y}{\alpha}\right]\left(\frac{a_m}{r}\right)^{\omega}+\frac{\rho_0\dot\lambda_p}{r^{n-1}}\left\{\frac{1}{\omega-n+1}\left[1-\left(\frac{a_m}{r}\right)^{\omega-n+1}\right]-\frac{\Lambda}{2n-\omega}\left(\frac{a_m}{r}\right)^{\omega-n+1}\right\}. \quad (2.3)$$

The expression obtained for $\omega-n+1=0$, i.e., $\alpha=\alpha_0=(1-2\Lambda)/(1+4\Lambda)$, goes over into the solution

$$\sigma_r(r)=\frac{Y}{\alpha}-\left[p(a_m)+\frac{Y}{\alpha}\right]\left(\frac{a_m}{r}\right)^{\omega}+\frac{\rho_0\dot\lambda_p}{r^{\omega}}\left(\ln\frac{r}{a_m}-\frac{\Lambda}{n+1}\right). \quad (2.4)$$

Therefore, the classification a) $0<\alpha<\alpha_0$; b) $\alpha=\alpha_0$; c) $\alpha>\alpha_0$ should be introduced in a medium with dilatancy instead of the classification [6]. As is seen from (2.3) and (2.4), radial stresses in the plastic domain can achieve a maximum at the point r_*:

$$r_*=a_m\left\{\frac{\omega}{n-1}-\frac{(\omega-n+1)\,\omega}{(n-1)\rho_0|\dot\lambda_p|}\left[p(a_m)+\frac{Y}{\alpha}\right]a_m^{n-1}+\frac{(\omega-n+1)\Lambda\omega}{(n-1)(2n-\omega)}\right\}^{1/(\omega-n+1)}$$

or

$$\ln\frac{r_*}{a_m}=\frac{1}{\omega}+\frac{\Lambda}{n+1}-\frac{\left[p(a_m)+\frac{Y}{\alpha}\right]a_m^{\omega}}{\rho_0|\dot\lambda_p|},\ \alpha=\alpha_0.$$

Let us use the "juncture" condition (1.19) for the solution (2.2) in the elastic domain and the solution (2.3) in the plastic domain to estimate the component containing $|\dot\lambda_p|$.

3. Let us first examine brittly fractured rock. Let us consider fracture to occur if the maximal compressive stress reaches σ_*. For such rock it can be considered that $Y\ll\sigma_*$. Taking into account that the equality $-\sigma_r(R_m)=\sigma_*$ should be satisfied at the time of stopping the cavity, we obtain an equation for R_m and $\rho_0|\dot\lambda_p|$:

$$\sigma_*=\left[p(a_m)+\frac{Y}{\alpha}\right]\left(\frac{a_m}{R_m}\right)^{\omega}-\frac{Y}{\alpha}-\left\{\sigma_*-p_h-\frac{4Ea_m^3}{9R_m^3}\left[\left(\frac{R_m}{a_m}\right)^{3\Lambda}-\left(\frac{a_0}{a_m}\right)^3\right]\right\}\left\{\frac{1}{\omega-n+1}\left[1-\left(\frac{a_m}{R_m}\right)^{\omega-n+1}\right]-\frac{\Lambda}{2n-\omega}\left(\frac{a_m}{R_m}\right)^{\omega-n+1}\right\};\quad (3.1)$$

$$\frac{Y}{\alpha}+p_h-\left[p(a_m)+\frac{Y}{\alpha}\right]\left(\frac{a_m}{R_m}\right)^{\omega}+\frac{4Ea_m^3}{9R_m^3}\left[\left(\frac{R_m}{a_m}\right)^{3\Lambda}-\left(\frac{a_0}{a_m}\right)^3\right]$$
$$=\rho_0|\dot{\lambda}_p|R_m^{n-1}\left\{1+\frac{1}{\omega-n+1}\left[1-\left(\frac{a_m}{R_m}\right)^{\omega-n+1}\right]-\frac{\Lambda}{2n-\omega}\left(\frac{a_m}{R_m}\right)^{\omega-n+1}\right\}. \quad (3.2)$$

Since $Y \ll \sigma_*$, i.e., adhesion in the fractured rock is small compared to the strength of the unfractured rock, it can then not be taken into account. Let us write the solution of (3.1) and (3.2) as well as the expressions for r_*, r_{**}, V_p for different values of ω:

1) $\omega - n + 1 > 0$

$$R_m = a_m\left\{\frac{4E}{9\sigma_*\left[(\omega-n+2)-\frac{p_h}{\sigma_*}\right]}\right\}^{1/3(1-\Lambda)}; \quad (3.3)$$

$$\frac{\rho_0|\dot{\lambda}_p|}{R_m^{n-1}} = (\omega-n+1)\sigma_*, \quad (3.4)$$

$$r_* = a_m\left\{\frac{\omega}{n-1}-\frac{\omega a_m^{n-1}p(a_m)}{(n-1)\sigma_* R_m^{n-1}}+\frac{(\omega-n+1)\Lambda\omega}{(n-1)(2n-\omega)}\right\}^{1/(\omega-n+1)},$$

$$r_{**} = \left[\frac{4Eg(R_m)}{3R_m\sigma_*}\frac{\omega-n+2}{\omega-n+1}\right]^{1/2},$$

$$V_p = \frac{9\sigma_* R_m^3\left[(\omega-n+2)-\frac{p_h}{\sigma_*}\right]}{4E}-a_m^3.$$

As is seen from (3.3), the radius of the fractionation zone diminishes with the rise in strength of the shattered rock in this case. This effect is determined entirely by the influence of the dynamics of the motion of the medium in the neighborhood of the cavity on the nature of the stress state. Let us note that the rise in lithostatic pressure results in the growth of R_m/a_m (if the dependence of σ_* on the depth is not taken into account). Let us also note that acceleration of the reverse motion, defined by (3.4), grows with the rise in strength of the shattered rock, and this effect is apparently related to the growth of the gradient of radial stresses with the rise of ω:

2) $\omega - n + 1 < 0$

$$R_m = a_m\left[\frac{4E}{9\sigma_*\left(1-\frac{p_h}{\sigma_*}\right)}\right]^{1/3(1-\Lambda)}; \quad (3.5)$$

$$\frac{\rho_0|\dot{\lambda}_p|}{R_m^{n-1}} = \sigma_*(n-\omega-1)\left(\frac{a_m}{R_m}\right)^{n-\omega-1}, \quad (3.6)$$

$$r_* = a_m\left\{\frac{\omega}{n-1}+\frac{\omega a_m^{n-1}p(a_m)}{(n-1)\sigma_* R_m^{n-1}}\left(\frac{R_m}{a_m}\right)^{n-\omega-1}-\frac{(n-\omega-1)\Lambda\omega}{(n-1)(2n-\omega)}\right\}^{-1/(n-\omega-1)}$$

$$r_{**} = \left[\frac{4Eg(R_m)}{3(n-\omega-1)R_m\sigma_*}\left(\frac{R_m}{a_m}\right)^{n-\omega-1}\right]^{1/2};$$

$$V_p = \frac{9\sigma_* R_m^3(1-p_h/\sigma_*)}{4E}-a_m^3. \quad (3.7)$$

In this case the radius of the pulverization zone is independent of the strength of the shattered rock and agrees with the quasistatic estimate [5]. Let us also note that acceleration of the reverse motion drops with the diminution in ω since the main dependence on ω in (3.6) is governed by the quantity $(a_m/R_m)^{n-\omega-1}$;

3) $\omega = n - 1$

$$\frac{\rho_0|\dot{\lambda}_p|}{R_m^{n-1}} = \frac{\sigma_*}{\ln\frac{eR_m}{a_m}}, \quad \ln\frac{r_*}{a_m} = \frac{1}{\omega}+\frac{\Lambda}{n-1}-\frac{a_m^{n-1}p(a_m)\ln\frac{eR_m}{a_m}}{\sigma_* R_m^{n-1}},$$

$$r_{**} = \left[\frac{4Eg(R_m)}{3R_m\sigma_*}\ln\frac{eR_m}{a_m}\right]^{1/2}.$$

The expressions for R_m and V_p are analogous to (3.5) and (3.7), respectively. Let us present estimates which show how much certain quantities change because of dilatancy by considering that Λ varies in an interval between 0.1 and 0.2. Thus, the radius r_* increases by 5.5-11%, the peak stress $\sigma_r(r_*)$ by 3-6%, the radius of the pulverization zone by 18-36%.

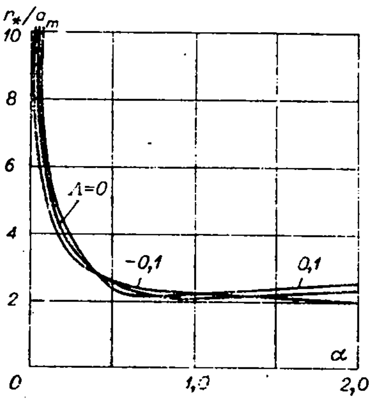

Fig. 3

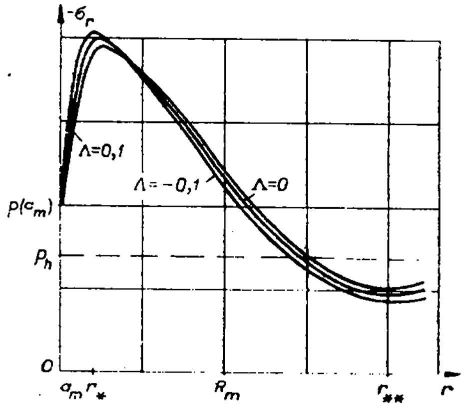

Fig. 4

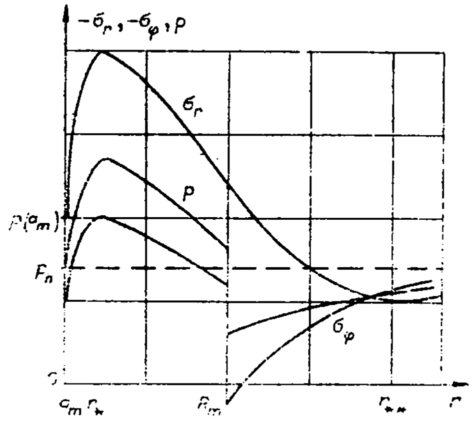

Fig. 5

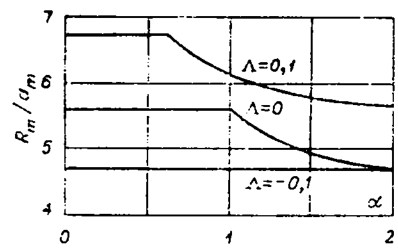

Fig. 6

A more detailed dependence of r_* on the parameters α and Λ is presented in Fig. 3. It is seen that dilatancy affects the pinch radius r_* slightly. At the same time, the change in strength of the pulverized medium substantially affects the quantity r_* in the domain of small α ($\alpha < 0.6$). In the domain of large α ($\alpha > 0.6$), the dependence of r_* on α is quite weak.

The qualitative behavior of $-\sigma_r(r)$ at the time the cavity stops is presented in Fig. 4 for different values of Λ and $\alpha = \alpha_0$. It is seen that a change in Λ does not result in a qualitative change in the nature of the stress state. For a more detailed characteristic of the stress state, graphs of $-\sigma_r(r)$, $-\sigma_\varphi(r)$, and $p(r) = -(1/3)(\sigma_r + 2\sigma_\varphi)$ are presented in Fig. 5 for $\Lambda = 0$, $\omega = 1$. It is seen that in the plastic domain all these quantities have a maximum, where the peak stress is sufficiently close to the cavity ($r_* \sim 2a_m$). The stress $-\sigma_\varphi(r)$ experiences a jump at the boundary of the rupture zone and becomes tensile. Therefore, there exists the possibility of radial crack formation. The influence of radial cracks is not taken into account in this paper, hence the results obtained are applicable for sufficiently large depths. Such a nature of the stress is associated with taking account of the dynamic terms. Thus, the pressure in the elastic domain becomes less than the lithostatic pressure by the quantity $\rho_0 |\dot{\lambda}_e|/r$.

The dependences of the radius R_m of the pulverization zone on the parameters α and Λ are presented in Fig. 6. It is seen that in this case the magnitude of the dilatancy substantially influences the quantity R_m. A change in dilatancy can hence result in a change in the qualitative dependence of the radius of pulverization due to the properties of the pulverized rock.

4. Let us examine the case $\alpha = 0$, i.e., an ideally plastic medium. The Coulomb–Mohr fluidity condition (1.1) goes over into the Tresk condition

$$\sigma_\varphi - \sigma_r = Y. \tag{4.1}$$

Taking account of (1.16) and (1.18), the solution of (1.12) has the form

$$\sigma_r(r) = -p_h - \frac{4E}{9}\,\mathrm{Li}_2\left(\frac{a^3 - a_0^3}{r^3}\right) - \rho_0\left(\frac{\dot{\lambda}_e}{r} - \frac{\lambda_e^2}{2r^4}\right).$$

Taking account of condition (1.10), we write in the plastic domain

$$\sigma_r(r) = -p(a) + 2Y\ln\frac{r}{a} + \rho_0\dot{\lambda}_p\left(\frac{1}{a} - \frac{1}{r}\right) - \frac{\rho_0\lambda_p^2}{2}\left(\frac{1}{a_4} - \frac{1}{r^4}\right).$$

Let us use the condition (1.19) on the rupture front at the time the cavity stops ($\dot{\lambda}_e = \dot{\lambda}_p = \dot{\lambda}$), then

$$-p_h - \frac{4E}{9}\frac{a_m^3 - a_0^3}{R_m^3} + \frac{\rho_0|\dot{\lambda}|}{a_m} = -p(a_m) + 2Y\ln\frac{R_m}{a_m}.$$

Using the fact that $(a_0/a_m)^3 \ll 1$, we write the solution in the plastic domain

$$\sigma_r(r) = -p(a_m) + 2Y\ln\frac{r}{a_m} - \left(1 - \frac{a_m}{r}\right)\left(2Y\ln\frac{R_m}{a_m} - p(a_m) + p_h + \frac{4E}{9}\left(\frac{a_m}{R_m}\right)^3\right). \quad (4.2)$$

As is seen from (4.2), the radial stresses can reach a maximum at the point r_*:

$$r_* = \frac{a_m}{2Y}\left[2Y\ln\frac{R_m}{a_m} + p_h - p(a_m) + \frac{4E}{9}\left(\frac{a_m}{R_m}\right)^3\right]. \quad (4.3)$$

Using (1.16) and (4.1) results in expressing the radius of the rupture zone in an ideal plastic medium as

$$R_m = a_m\left(\frac{2}{3}\frac{E}{Y}\right)^{1/3}. \quad (4.4)$$

Substituting (4.4) into (4.3), we obtain

$$\frac{r_*}{a_m} = \frac{1}{3}\left[\ln\left(\frac{2}{3}\frac{E}{Y}\right) + 1\right] + \frac{p_h - p(a_m)}{2Y}. \quad (4.5)$$

As estimates from [6] show, for small p_h we have $p(a_m) - p_h > 0$. Taking into account here that $Y/E \sim 10^{-3}$ for ideal plastic media, the radius r_* turns out to be less than $2.5a_m$; i.e., the maximal stresses will be near the cavity or on its boundary. As the lithostatic pressure grows $p(a_m) - p_h$ decreases and can change sign. In this case r_* grows, and the domain of maximal compressive stresses is shifted from the cavity.

We obtain for the maximal compressive stresses from (4.2) and (4.5)

$$-\sigma_r(r_*) = p(a_m) + 2Y\left[\frac{r_*}{a_m} - \ln\frac{r_*}{a_m} - 1\right].$$

Thus for $r_* \approx 2.5a_m$ (for $p_h \sim p(a_m)$) the maximal stress exceeds the pressure in the cavity by a quantity $\approx 1.5Y$.

Analysis of the stress state in the neighborhood of an expanding cavity at the time of stopping permits noting the following:

1. Pinching exists in a dilating medium with dry friction for $\alpha > \alpha_*$. A dependence of α_* on Λ is obtained: $\alpha_* = \alpha_*^0 + 0.1\Lambda$ ($\alpha_*^0 = 0.06$; $\varphi_*^0 = 1.5°$). However, as is indicated in [6, 7] $\varphi \approx 12$-$30°$ in real soils with friction. Figure 3 shows that for these angles the dependence of r_* on α is sufficiently weak, and $r_* \approx 2a_m$. The peak stress $-\sigma_r(r_*)$ moves from the cavity under loosening and diminishes; under compression a shift towards the cavity occurs and the peak is increased.

2. The radius R_m of the pulverization zone and the minimum in the elastic domain r_{**} depend substantially on ω. Under loosening, R_m and r_{**} increase, while under compression they decrease.

3. In the case of plastic media, the zone of maximum compressive stresses turns out to be near the cavity for moderate lithostatic pressures. The peak stress moves away from the cavity as the lithostatic pressure grows.

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MODEL OF THE SOIL AND COMPUTATIONAL COMPLEX FOR THE ANALYSIS OF UNDERGROUND EXPLOSIONS

V. V. Bashurov, Yu. S. Vakhrameev, S. V. Dem'yanovskii, V. V. Ignatenko, and T. V. Simonova

UDC 518.12:539.3

In underground explosions executed in the interests of ejection, downcomer funnel or bulging hillock formation, the soil properties influence not only the quantitative parameters substantially, but also the qualitative pattern of the explosion. Thus, under the same conditions of charge embedding and power, a downcomer funnel or bulging hillock can be formed depending on the properties of the rock. The majority of explosions are performed in hard rock. Hence, the model of the soil should be suitable to describe its fundamental properties. A model of rocky soil is presented in this paper, the scheme for a numerical computation of the problem is described, and results of certain computations are presented.

1. An unruptured medium is considered elastic. Rupture sets in instantaneously upon the attainment of definite criteria. Right after rupture, which occurs in brittle material under insignificant strains, the pulverized rock consists of separate compactly contiguous pieces. In this state its volume density is 1.5-1.7 times greater than in rubble fill. The compact fractured medium and loose rubble differ quite radically in the effective value of the internal friction and ~100 times in the volume compressibility. It is hence important to take account of the gradual change in the properties of the ruptured medium as it loosens.

In both states, before and after rupture, the medium is considered isotropic. The pressure and degree of looseness are taken as parameters of the state in this model. The influence on the mechanical properties of the size of the pieces, their shape, and temperature are neglected.

The equations of state of a ruptured medium are written in differential form. The change in density is defined by the equation

$$d\rho/\rho = dp/K_{1,2}(p, \rho) - \Phi(p, \rho)\sqrt{J_2}dt, \tag{1.1}$$

where J_2 is the second invariant of the strain rate deviator.

The first term on the right corresponds to pure volume strain, while the second describes the dilatancy effect. An analogous equation was examined in [1] in application to friable media and soft soils. In this paper (1.1) is extended to all states of ruptured rocky soil, including rubble, and a state with dense packing. Hence, $K_{1,2}$ and Φ are understood to be strongly varying functions of their arguments. The irreversibility of the volume strain is taken into account by the fact that the absolute value of the volume compressibility $K_{1,2}$ depends on the sign of dp. In the construction of the function $\Phi(p, \rho)$ it is assumed for simplicity that:

a) for $dp = 0$ the shear strains result only in loosening of the substance;

b) the loosening intensity vanishes at the extreme curve $p_2(\rho)$ (Fig. 1) corresponding to monotonic compression of the loosest rubble. The domain of possible states of the ruptured medium is shown in Fig. 1.

Chelyabinsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 153-160, May-June, 1979. Original article submitted December 27, 1977.

 0021-8944/79/2003-0380$07.50